

Portfolio Selection with The Index Model on Constraints

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Abstract: The construction of portfolios is crucial to attaining stable returns in finance. This experiment will construct portfolios using 20 years of historical price data from well-known firms in industries across the financial market, an equity index, and a proxy for the risk-free rate. This research paper will use the Index Model to establish optimization inputs for calculating portfolios while considering existing regulations and risk aversion. Optimization results of the portfolios will be represented in data and graphical forms and discussed in greater detail. The experiment will then analyze the optimization performance in each situation and obtain the patterns retained for future studies.

1. Introduction

One of the goals of financing is to maximize one's profit while minimizing the risk of achieving so. Investment of a single instrument may be fruitful but suffers greatly from the reliability of achieving the desired return. On the other hand, investing in combined assets will diversify risk from one instrument, thus reducing your overall risk, known as an investment portfolio. The Modern Portfolio Theory is an investment strategy study that allocates asset investments to maximize the expected return while minimizing risks.

Modern Portfolio Theory was at the stage of rapid development in the 1960s as a scientific way of studying financial activities. In 1952, Harry Markowitz pioneered the modern portfolio theory by applying mathematical calculation to maximize the overall return between an optimal amount of risk and return based on risk tolerance [1]. In 1963, William F. Sharpe developed the Single Index Model based on Markowitz's modern portfolio theory, which greatly reduces and simplifies the calculation that increases the practicality in comparison [2]. Between 1964 to 1966, Sharpe [3], Linter [4], and Mossin [5] developed the widely accepted capital asset, pricing model.

B.H. Solnik applied the Index model in the international market to analyze the effect of domestic factors and international factors on asset prices.

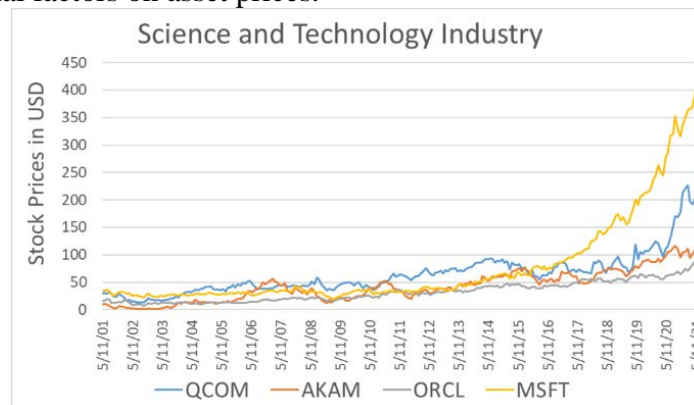


Figure 1. Science and Technology Industry prices

Our experiment will utilize the Index Model to construct portfolios of some of the most leading forces from several industries across the financial market. The experiment is aimed to construct portfolios that imitate some realistic aspects of the financial market, such as regulations and clients'

risk preferences. The portfolio construction will produce the optimized result for each unique situation. The experiment will then analyze the performance of the optimization.

The organization of the research will be as follow: section 2 goes into details of the data and instrument background, section 3 explains the index mode, constraints, and risk preference in greater detail, and section 4 produces the optimized results and analyze the performance of our portfolio in situations.

2. Data

A well-diversified portfolio may consist of investment combinations such as stocks, bonds, and securities, across the financial market in different industries. To illustrate our research, we aggregated 20 years of historical daily return data extracted from Bloomberg, from May 11 of 2001 to May 12 of 2021, of ten different stocks that are leaders across three financial industries, an equity index (Standard and Poor's 500), and a proxy for risk-free rate (Fed Funds rate).

We have four stocks in the Science and Technology Industry. The first stock is QUALCOMM Incorporated (QCOM), a wireless technology corporation specializing in semiconductors, software, and wireless technology services. It owns some of the most critical patents in mobile communication standards. The second stock is Akamai Technologies, Inc (AKAM), a content delivery network provider specializing in cybersecurity and cloud service. The third stock is Oracle Corporation (ORCL), a software company specializing in database software and technology, cloud system, and enterprise software. The fourth stock is Microsoft Corporation (MSFT), specializing in computer software, consumer electronics, and personal computers. The stock prices for the science and technology industry stocks over 20 years are displayed in **Figure 1**:

We have three stocks in the Energy Industries. The first stock is Chevron Corporation (CVX), an energy corporation specializing in exploration, production, refining, marketing, transportation, and sale of crude oil and natural gas. The second stock is Exxon Mobil Corporation (XOM), an energy corporation specializing in exploration, production, trade, transportation, and sale of crude oil and natural gas. The first stock is Imperial Oil Limited (IMO), specializing in producing crude oil, diluted bitumen, and natural gas. The stock prices for the energy industry stocks over 20 years are displayed in **Figure 2**.

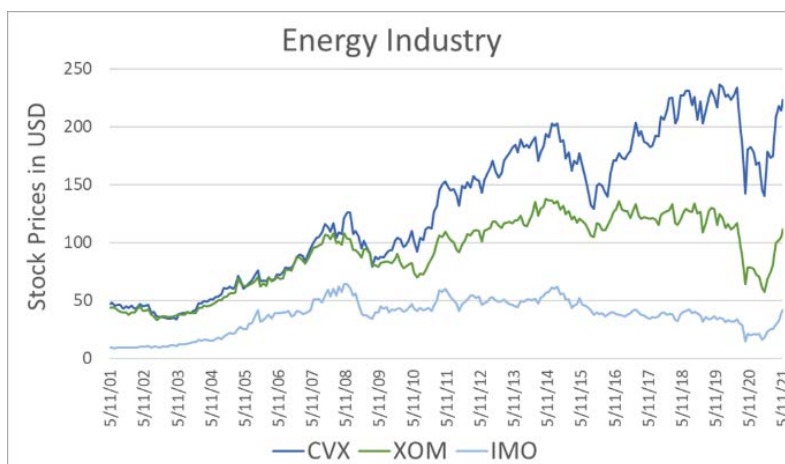


Figure 2 Energy Industry prices

We have three stocks from Beverage and Food Manufacturing Industry. The first stock is the Coca-Cola Company (KO), a beverage corporation specializing in manufacturing, retailing, and marketing non-alcoholic beverage concentrates, syrups, and alcoholic beverages. The second stock is PepsiCo, Inc (PEP), a food, snack, and beverage corporation specializing in manufacturing, distribution, and marketing its wide range of food and beverage brands products. The third stock is McDonald's Corporation (MCD), a fast-food company specializing in manufacturing, marketing, and sale of their products. The stock prices for the beverage and food manufacturing industry stocks over 20 years are displayed in **Figure 3**.

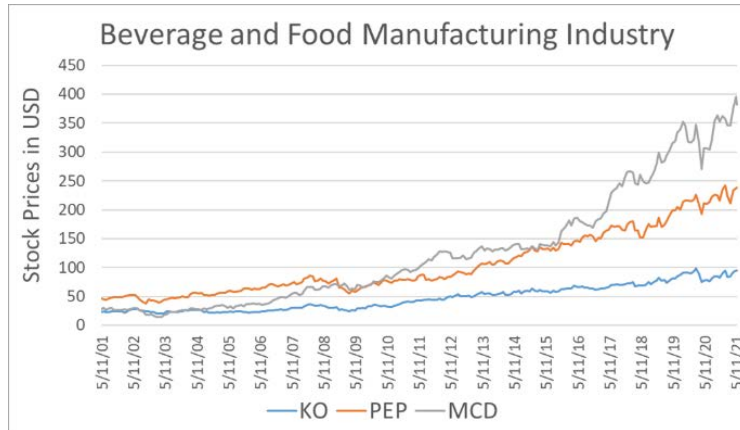


Figure 3. Beverage and Food Manufacturing Industry prices

Furthermore, we choose to include an equity index and a proxy for a risk-free rate to diversify the instrument lineup of our portfolio. Our index of choice is the Standard and Poor’s 500 (SPX), as it is a well-known index that tracks the performance of 500 large companies and thus represents the overall health of the stock market. Our proxy for the risk-free rate is the one-month annual Federal Funds Rate (ticker: FEDL01).

A Nominal Risk-Free Rate (NRFR) is the yield on a risk-free asset without the effect of inflation. NRFR is crucial for calculating the excess return in the later steps. The risk-free asset we refer to is the federal funds rate in our model. Assuming that there are 252 working days in a year, let RFR_{-1} represent the risk-free return from the previous day, and give the first day of risk-free return a placeholder value of 1. A fictional risk-free return (RFR) can be computed by:

$$RFR = RFR_{-1} \cdot \left(\frac{1 + \frac{FEDL01}{100}}{252} \right) \quad (1)$$

When compared with the overarching trend, working with daily prices will come with unwanted price fluctuation outliers. To reduce the outliers in our data set, which reduces the non-Gaussian effect, we first have to convert daily prices to monthly prices. We can achieve this by creating a new data set with only the stock prices and RFR of the last day of each month.

Let $NRFR_{-1}$ represent the nominal risk-free rate of the previous month. The monthly NRFR can be computed by:

$$NRFR = \left(\frac{RFR}{RFR_{-1}} \right) - 1 \quad (2)$$

We can conduct a correlation table to understand our portfolio’s instrument components further. The Data Analysis ToolPak add-in in Excel offers a Correlation function that allows us to input a series of data set and return us with a complete data correlation table. A portfolio with a higher correlation will undertake higher risks, as returns between each stock will be more likely to perform similarly. In contrast, a less correlated portfolio with diversified stocks will be more likely to disperse risks and reduce the overall volatility. The correlation table can be observed in **Table 1**.

Table 1 Correlation Table

Corr.	SPX	QCOM	AKAM	ORCL	MSFT	CVX	XOM	IMO	KO	PEP	MCD
SPX	1	0.557	0.389	0.546	0.639	0.613	0.568	0.522	0.491	0.522	0.537
QCOM	0.557	1	0.278	0.285	0.375	0.233	0.235	0.272	0.197	0.263	0.262
AKAM	0.389	0.278	1	0.242	0.256	0.122	0.068	0.127	0.085	0.102	0.291
ORCL	0.546	0.285	0.242	1	0.475	0.264	0.301	0.233	0.068	0.205	0.137
MSFT	0.639	0.375	0.256	0.475	1	0.339	0.304	0.250	0.279	0.334	0.357
CVX	0.613	0.233	0.122	0.264	0.339	1	0.829	0.734	0.402	0.272	0.394
XOM	0.568	0.235	0.068	0.301	0.304	0.829	1	0.697	0.338	0.240	0.340
IMO	0.522	0.272	0.127	0.233	0.250	0.734	0.697	1	0.297	0.178	0.268
KO	0.491	0.197	0.085	0.068	0.279	0.402	0.338	0.297	1	0.579	0.499
PEP	0.522	0.263	0.102	0.205	0.334	0.272	0.240	0.178	0.579	1	0.470
MCD	0.537	0.262	0.291	0.137	0.357	0.394	0.340	0.268	0.499	0.470	1

3. Method

Our experiment aims to construct an optimal portfolio using the index model, discussed in 3.1 and 3.2, complying with various situations. The situation can be composed of the client’s constraints and risk-aversion. Constraints will be discussed in 3.3, and risk-aversion will be discussed in 3.4.

3.1. Index Model

The Index Model (IM), also known as the Single-Index Model, is an asset pricing model created by William F. Sharpe in 1963. The Index Model is simpler than the Markowitz Model, which requires much larger estimators to analyze the risk and return. The Index Model suggests that the risk and return of a stock can be decomposed into the systematic factor on a macroeconomic scale ($\beta_i R_m$), expected firm-specific factor on a microeconomic scale (α_i), and unexpected firm-specific factors on a microeconomic scale (ε_i). The systematic factor, such as a nationwide increase of minimum wage, can simultaneously affect multiple firms on various scales. The firm-specific factors, such as the resignation of key personnel, can affect the firm itself.

The excess return formula of the Index Model is expressed as mathematical terms below:

$$R_i = \beta_i R_m + \alpha_i + \varepsilon_i \quad (3)$$

Where:

$$R_i = r_i - r_f \quad (4)$$

$$R_m = r_m - r_f \quad (5)$$

In words, the stock i 's excess return (R_i) is equal to the responsiveness to the market (β_i) times the excess return of the market (R_m), plus the stock i 's outperformance compared to the market (α_i), plus stock i 's surprise return or residual (ε_i).

Note that the excess return of stock i (R_i) is equal to return of stock i (r_i) minus risk-free rate (r_f), and excess return of the market (R_m) is equal to return of the market (r_m) minus risk-free rate (r_f). Also, note that the residual (ε_i) can be interpreted as a statistical error that is normally distributed with a mean of zero and standard deviation of σ_i in mathematical terms.

3.2. Index Model Return, Standard Deviation, and Sharpe Ratio

Before we get into the index model's return, standard deviation, and Sharpe ratio, a few key optimization inputs need to be defined as they play a crucial role in the model calculation.

Let p_i represent the price of the current month and p_{i-1} represent the price of last month, the monthly return (r_i) of each stock and index can be computed by:

$$r_i = \frac{p_i}{p_{i-1}} \quad (6)$$

With 12 months in a year, the monthly annualized average excess return (μ_i) for each stock and index can be computed by:

$$\mu_i = 12 \cdot \frac{1}{n} \sum_{i=1}^{11} R_i \quad (7)$$

The weight of a stock or index in a portfolio can be represented by w_i . The symbol \vec{w} and $\vec{\mu}$ represent the weight composition and annualized average excess return of all instruments in the portfolio in vector form.

Finally, with the inclusion of all previous considerations, the mathematical formulas of portfolio return (r_p), standard deviation (σ_p), and Sharpe ratio (S_p) of the index model can be expressed as below:

Return:

$$r_p = \vec{w} \cdot \vec{\mu} \quad (8)$$

Standard deviation or risk:

$$\sigma_p = \sqrt{(\sigma_M \beta_p)^2 + \sum_{i=1}^{11} w_i^2 \sigma^2(\varepsilon_i)} \quad (9)$$

Where:

$$\beta_p = \vec{w} \cdot \vec{\beta} \quad (10)$$

Sharpe ratio measures the return performance of the portfolio compared to its risk. Sharpe ratio:

$$S_p = \frac{r_p}{\sigma_p} \quad (11)$$

3.3. Constraints

We have to consider policies and regulations in finance when implementing the index model. When modeling the optimizations, the policies and regulations can be translated into mathematical constraints to our equations. We are implementing five constraints under each of their situations as the following describes:

The first constraint is designed to simulate Regulation T by FINRA, allowing broker-dealers to allow their customers to have 50% or more of the positions funded by the customer's account equity. This constraint can be expressed as:

$$\sum_{i=1}^{11} |w_i| \leq 2 \quad (12)$$

The second constraint is designed to simulate the arbitrary “box” constraints on the weights of each instrument in a portfolio. This constraint can be expressed as:

$$|w_i| \leq 1, \text{ for } \forall i \quad (13)$$

The third constraint is designed to simulate the “free” problem. It illustrates how the area of permissible portfolios in general and the efficient frontier, particularly, looks like if no additional optimization constraints are applied.

The fourth constraint is designed to simulate the typical limitations in the U.S. mutual fund industry, where open-ended mutual funds are not allowed to have any short positions. This constraint can be expressed as:

$$w_i \geq 0, \text{ for } \forall i \quad (14)$$

The fifth and final constraint is designed to simulate the inclusion of the broad index to our portfolios and test whether that brought a positive or negative impact. This constraint can be expressed as:

$$w_1 = 0 \quad (15)$$

3.4. Risk-Aversion

Risk-aversion is the client’s preference of risk that comes with levels of return. Most preferences can be sorted as risk-averse or risk-loving [6-8]. A risk-averse client will prioritize to minimize risk over maximize return. On the other hand, a risk-loving client will seek a greater return and overlook the higher risk from the return. The experiment will consider these risk preferences and provide the client with two potential portfolio options: the minimum risk portfolio and the efficient risk portfolio.

The minimum risk portfolio (*MRP*), also known as the global minimum variance portfolio, is the result of a stock weight combination that minimizes the portfolio variance (σ_p^2), which can be expressed as:

$$\text{MRP} = \min \sigma_p^2 S_p = \frac{r_p}{\sigma_p} \quad (16)$$

The efficient risk portfolio (*ERP*), also known as the maximal Sharpe ratio, is the result of a stock weight combination that maximizes the Sharp ratio (S_p), which can be expressed as:

$$\text{ERP} = \max S_p \quad (17)$$

Be sure the symbols in your equation have been defined before the equation appears or immediately follows. Please refer to "Equation (1)," not "Eq. (1)" or "equation (1)."

4. Result Analysis

As discussed in part 3, our calculation will output an *MRP* and an *ERP* for each of the five constraints. On top of that, a graph containing risk frontiers that will visualize our investment decision is provided for each of the five constraints.

4.1. Optimal Portfolio

MRP and *ERP* can be calculated by utilizing the solver add-in program in excel. The solver program allows users to either maximize or minimize an objective with a set of variables and certain constraints [9]. The program will then calculate the maximized or minimized objective value and provide the variable results that lead to the objective value.

In the case of computing *MRP*, portfolio standard deviation (σ_p) will be the objective to be minimized. Similarly, in the case of *ERP*, Sharp ratio (S_p) will be the objective to be maximized. The weight of each stock and index will be the changing variable.

The program allows for constraints to the calculation, where we input each constraint and repeat all previous steps till all five constraints received one *MRP* and one *ERP*.

The final result for the optimal portfolio will be:

The result guides the investment proportion for each of the instruments in the portfolio, which achieves our desired amount of expected return, risk, and Sharpe ratio.

The main goal of *MRP* is to invest in the instruments that offer low volatility and high stability, which is why results in *MRP* show us a consistent level of low risk (standard deviation) across the five constraints. Instruments such as SPX, CVX, XOM, KO, PEP, and MCD show positive weights that indicate investments with steady growth, thus worth long positioning. In those positive weighted instruments, all three stocks in the beverage and food manufacturing industry held the largest component to our portfolio, which reflected that those stocks in this industry are generally more stable than the others in our portfolio. Instruments such as QCOM, AKAM, ORCL, MSFT, and IMO mostly show negative weights, indicating that they are investments with volatile growth, thus being short-positioned. All stocks in the technology and science industry had a negative weight, indicating they were unstable compared to other industries and the index in our portfolio [10].

Table 2 Minimum Risk Portfolios

MRP	SP X	QC OM	AK AM	OR CL	MS FT	CV X	XO M	IM O	KO	PE P	MC D	Ret urn	StD ev.	Shar pe
Cons tr1	0.1 29	- 0.04 2	- 0.02 5	- 0.00 5	- 0.00 1	0.0 33	0.0 91	- 0.01 4	0.2 98	0.3 67	0.1 69	0.07 5	0.11 4	0.66 4
Cons tr2	0.1 29	- 0.04 2	- 0.02 5	- 0.00 5	- 0.00 1	0.0 33	0.0 91	- 0.01 4	0.2 98	0.3 67	0.1 69	0.07 5	0.11 4	0.66 4
Cons tr3	0.1 29	- 0.04 2	- 0.02 5	- 0.00 5	- 0.00 1	0.0 33	0.0 91	- 0.01 4	0.2 98	0.3 67	0.1 69	0.07 5	0.11 4	0.66 4
Cons tr4	0.0 14	0.00 0	0.00 0	0.00 0	0.00 0	0.0 34	0.0 94	0.00 0	0.3 06	0.3 78	0.1 74	0.08 4	0.11 5	0.72 7
Cons tr5	0.0 00	- 0.03 4	- 0.02 3	0.00 5	0.01 6	0.0 49	0.1 06	- 0.00 5	0.3 14	0.3 87	0.1 85	0.07 9	0.11 4	0.69 2

Table 3 Efficient Risk Portfolios

ERP	SP X	QC OM	AK AM	OR CL	MS FT	CV X	XO M	IM O	KO	PE P	M CD	Ret urn	StD ev.	Shar pe
Cons tr1	- 0.4 84	0.04 4	0.06 2	0.06 5	0.21 7	0.0 44	- 0.0 16	0.0 38	0.2 05	0.3 35	0.4 89	0.13 6	0.14 5	0.93 7
Cons tr2	- 0.9 38	0.08 2	0.07 9	0.10 7	0.29 8	0.1 02	- 0.0 37	0.0 72	0.2 54	0.4 03	0.5 78	0.15 6	0.16 3	0.95 3
Cons tr3	- 0.9 38	0.08 2	0.07 9	0.10 7	0.29 8	0.1 02	- 0.0 37	0.0 72	0.2 54	0.4 03	0.5 78	0.15 6	0.16 3	0.95 3
Cons tr4	0.0 00	0.00 6	0.05 3	0.02 0	0.14 3	0.0 00	0.0 00	0.0 00	0.1 22	0.2 37	0.4 19	0.12 0	0.13 8	0.87 2
Cons tr5	0.0 00	0.01 7	0.05 8	0.03 3	0.16 9	- 0.0 14	- 0.1 38	0.0 10	0.1 44	0.2 67	0.4 53	0.12 9	0.14 6	0.88 5

The main goal of *ERP* is to invest in the instruments that offer the highest efficiency, which is why results in *ERP* show us a consistent high Sharpe ratio across the five constraints. Instruments such

as QCOM, AKAM, ORCL, MSFT, CVX, IMO, KO, PEP, and MCD show positive weights that indicate investments with efficient returns compared to the risk it takes. Of those positive weighted stocks, MSFT, KO, PEP, and MCD play the biggest component of the long positions, which indicates they have a return worth the risk. On the other hand, instruments such as SPX and XOM generally had a negative weight in the portfolio, indicating their poor return performance over risks.

The results show that constraints 1 and 2 had the most resemblance to constraint 3 compared with constraints 4 and 5. Thus the first two constraints are less impactful regulations to non-intervention decisions.

Constraint 4 shows us the most deviated result from constraint 3 by having the highest risks in *MRP* and lowest Sharpe ratio in *ERP* compared to other constraints. Even though constraint 4 could be beneficial to achieve higher return and Sharpe ratio in *MRP* and lower risk in *ERP*, it failed to achieve its optimization in minimizing or maximizing their objectives.

Constraint 5 had a different weight composition of instruments from the first three constraints because the index, which played a major role in any portfolio, is banned. However, the feasibility of short positioning, constraint 5 achieves better results than constraint 4 yet still underperformed by the first three constraints in achieving optimization objectives.

4.2 Risk frontiers

The risk frontiers are a line composed of the lowest possible amount of risk for any portfolio. The horizontal axis represents the standard deviation, and the vertical axis represents the expected return of a given point on the graph representing a possible portfolio. There are three major components of the risk frontier which are efficient frontier (*EF*), inefficient frontier (*IF*), and minimal risk frontier (*MRF*). *MRF* will generally follow a bullet shape with the head of the bullet facing left and two tails facing right. The *EF* lays on the upper half of the tail, and *IF* lays on the bottom half, separated by *MRP* that lays on the very tip of the bullet. *ERP* lays on the *EF* that has the largest slope of return over risk. The capital allocation line (*CAL*) is tangent to the *EF* on the *ERP* that represents all possible combinations of the risk-free assets to risky assets from the origin to *ERP*.

We will use the solver and solver table Excel add-ins to generate the data that plots the risk frontiers. A dummy variable is created to utilize our programs. To calculate *MRF*, we first set standard deviation as the object to minimize in the solver and set return equal to

the dummy variable as constraint, then run solver table with input cell equal to the dummy variable. To calculate *EF*, we set to return as the object to maximize in the solver and set standard deviation equal to the dummy variable as constraint, then run solver table with input cell equal to the dummy variable. To calculate *IF*, repeat the same steps as finding *EF* except we set to return as the objective to minimize.

The above process is repeated for every constraint by adding five additional constraints into the solver. After a brief while of program processing, the result of the graphs will look like

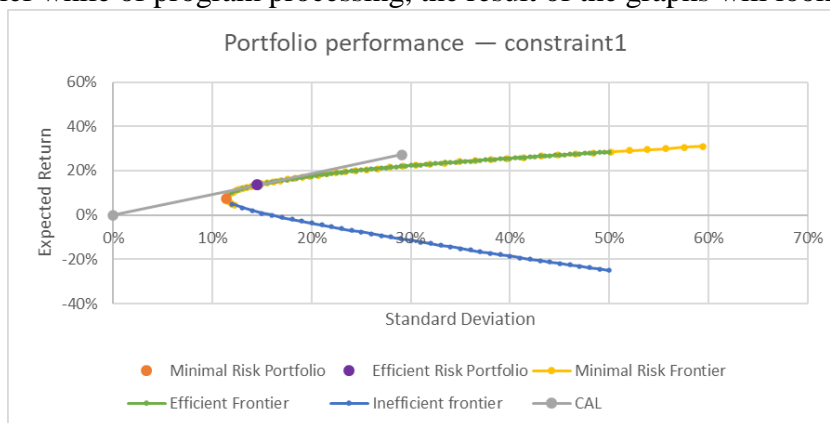


Figure 3. Portfolio performance under constraint 1

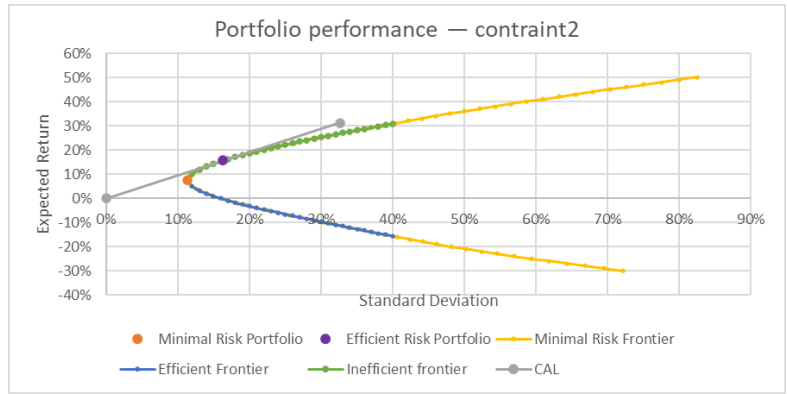


Figure 4. Portfolio performance under constraint 2

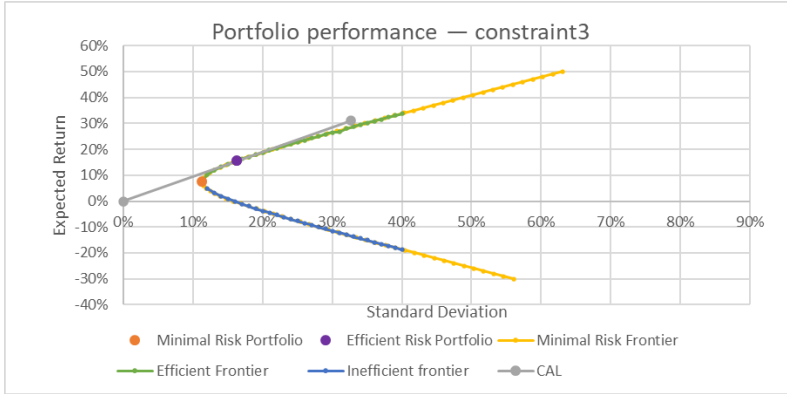


Figure 5. Portfolio performance under constraint 3

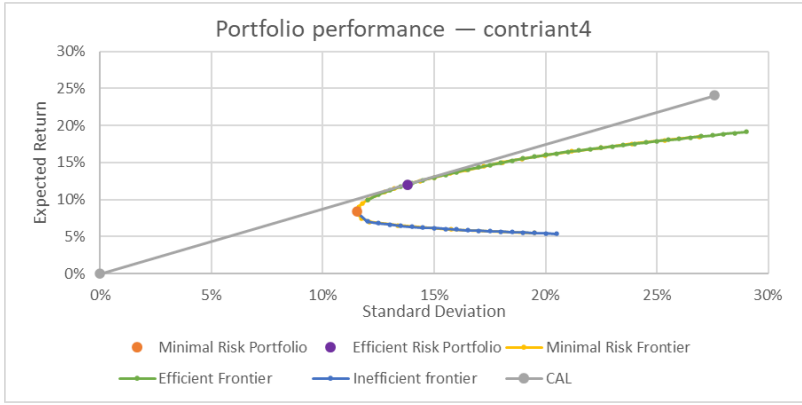


Figure 6. Portfolio performance under constraint 4

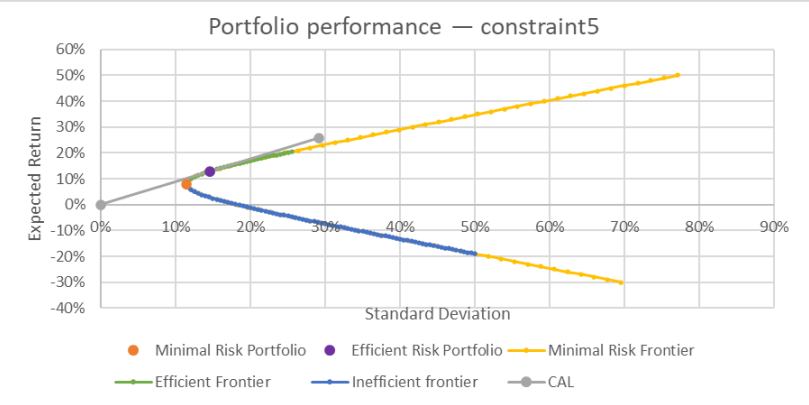


Figure 7. Portfolio performance under constraint 5

The graph visualization of risk frontiers and optimal portfolios portrays our interpretation of constraints' effects. The shapes of risk frontiers in Constraints 1 and 2 are not vastly different from constraint 3 as they suffer the least impact from regulations and become the distorted version of constraint 3. Constraint 5 suffers a bit more than constraints 1 and 2 and thus has more distorted frontiers with less Sharpe ratio variation, which is shown by a pointier tip and a more constant slope on the two tails. The most deviated result came from constraint 4, which had an extremely distorted curve that demonstrated the effect of no-shorting limitation on achieving optimal results.

5. Conclusion

In this experiment, we are interested in constructing optimal portfolios from a collection of instruments containing stocks, an index, and federal funds. Our experiment chooses the index model to establish portfolios because of the practicality of its calculation. Facing the complexity of the finance world, constructing different optimal portfolios under a combination of regulations and preferences is necessary in order to capture the interest of a broader audience of clients. We set five constraints to mimic some of the most common regulations and observe the effect. In each constraint, we also provide two optimal portfolios, minimal risk and efficient risk portfolios, to fit the client's risk preference. At the end of the experiment, a visualized version of our models, including optimal portfolios and risk frontiers, was established to further analyze the portfolio's performance under each constraint.

Constraint 3 served as a control group that does not suffer from any regulation burden for the sake of observing impact from other constraints. We found that Constraints 1 and 2 suffer relatively little as they performed similarly to constraint 3. Constraint 5 suffered more than the first two as it requires a drastically different portfolio composition. However, our experiment found a significant impact from constraint 4, which is supposed to mimic the regulation that does not allow short positions. Constraint 4 also drove the optimization objective furthest from any other constraints.

Although this experiment focuses on applying the index model in various situations with different clients, it suffers the lack of comprehensiveness from other aspects of simulating the real world. As mentioned earlier, we choose to utilize the index model because of its practicality but overlook the potential to decrease accurate performance when using other models such as the Full Markowitz Model. The priority of future experiments is to include the Full Markowitz Model and analyze the performance difference between the two models.

Another shortcoming of this experiment is the limited instrument selection, which led to the inadequacy of making possible claims about the growth of a certain industry. We will include a broader range of instrument selection on different scales of stocks to analyze the growth and potential of industries.

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